

Geometric buckling measurements using the pulsed neutron source method

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With 7 figures in the text

SUMMARY

The geometric buckling of cylindrical reactors with one or both ends rounded has been determined by pulsed source measurements with small polyethylene geometries. The results were in general accord with theoretical calculations. The diffusion parameters of polyethylene were also determined. The diffusion length was 2.12 ± 0.03 cm, and for the capture cross section of hydrogen a value of 0.337 ± 0.005 barns was found. The effect of control rods was studied using thin cadmium rods in water. Good agreement was found for axial, central rods, whereas the theoretical predictions for the effect of diagonal rods were too high.

Introduction

When the diffusion properties of a moderator are studied with the pulsed neutron source technique in a small geometry, the time decay constant of the fundamental mode of the neutron distribution is given by

$$\lambda = \lambda_a + D_0 B_g^2 - (C - d) B_g^4, \quad (1)$$

where λ_a is the absorption rate,
 D_0 is the diffusion constant,
 B_g^2 is the geometric buckling,
 C is the diffusion cooling coefficient, and
 d is a correction term [1].

The geometric buckling is defined as the lowest eigenvalue of the equation

$$\nabla^2 \phi + B^2 \phi = 0, \quad (2)$$

with the condition that the neutron flux vanishes at the effective boundaries of the body in question. By measurement of the time constant for various values of the buckling, which is easily calculated for simple shapes, the parameters λ_a , D_0 , and $C - d$ can be determined. For details of this type of measurements see reference 2.

Once these three parameters are known a measurement of the time constant can determine the buckling for a system of complex geometry. In spite of the

importance of geometric buckling in reactor theory this possibility has not been much investigated yet. The only measurements reported are those by Beckurts on the effect of cadmium rods in graphite [3]. In the work to be described here two different studies were made, one on moderators of special shapes and one on the effect of small absorbers corresponding to control rods in a reactor. Some preliminary results from these measurements have already been reported [4].

The equivalence between a reactor and a pulsed moderator

The question of how the results from pulsed experiments on a small moderator system can be applied to a critical reactor has been studied in an earlier report [1]. It was shown that if the time dependence is removed by the introduction of a fictitious capture cross section, the one-group equation for the pulsed model is identical with that for a critical reactor. The number of secondaries per collision and the total mean free path for the pulsed model are denoted c^* and l^* to indicate that they have been modified by the fictitious capture. The equivalence between a critical reactor and a pulsed model can then be expressed as follows:

The eigenvalues c and c^* are the same if

- (a) the neutron spectra in the two systems are independent of position,
- (b) the angular distribution of neutrons resulting from collisions is independent of velocity,
- (c) the eigenvalues c and c^* are close to unity,
- (d) the linear dimensions of the two systems are the same when measured in units of the respective transport mean free path.

Both for a critical reactor and a pulsed model the buckling is related to c and l through the formula

$$\frac{1}{c} = \frac{1}{B_g l} \tan^{-1} B_g l, \quad (3)$$

which is valid when the scattering is isotropic. When this is not so, more complicated relations can be found. However, as the one-group transport equation has the same form for both reactor and model, the formula connecting B_g^2 with l and c must also be equally valid for both cases. Thus within the limitations of one-group theory the geometric buckling measured with the pulsed method on a small moderator can be interpreted in terms of a critical reactor.

For the moderator the transport mean free path to be used in the comparison with the reactor is

$$l_{tr}^* = l^* / (1 - b_1), \quad (4)$$

Here b_1 is the average cosine of the scattering angle and l^* is given by

$$l^* = \frac{1}{\Sigma_s (1 - B_g^2 / 3 \Sigma_{tr} \Sigma_s)}. \quad (5)$$

Therefore, the transport mean free path will vary slightly with buckling. Such a variation can also arise from the average scattering cross section Σ_s , which may be influenced by the change in neutron spectrum caused by diffusion cooling. However, for the geometries considered here these corrections are so small that they can be neglected.

Method of measurement

The apparatus used in the experiments was essentially the same as in earlier measurements [5]. In spite of the heavy shielding for stray neutrons, the background was in general higher. This was caused by the enforced situation close to the reactor R1. The temperature during the measurements was 20°C.

As before, the first part of the decay curve after the burst was discarded to avoid the influence of neutrons not yet slowed down. The higher harmonic modes were suppressed by suitable arrangement of target, cadmium masks, and detectors, and corrections were made for the remaining effects. After the additional corrections for the deadtime of the counting equipment and for background the decay constant was calculated using the method of Peierls [6]. The best check on the results was the consistency between values obtained with different pulse lengths, repetition rates and detector positions.

Reactors of special shapes

The one-group equation for a bare, homogeneous, critical reactor (2) can be solved in eleven different orthogonal coordinate systems by separation of the variables [7], but the solutions cannot always be expressed in standard functions. The coordinate surfaces determine the reactor shapes which can be treated in the particular coordinate system. All these surfaces are of the second degree, so the number of reactor geometries which can be tackled is very low. For instance, the rather common reactor shape of a cylinder rounded at one or both ends cannot be analysed by separation of the variables. Our first studies were therefore devoted to this kind of geometry. More specifically the rounded ends were either spherical segments with height half the radius of the cylinder or hemispheres.

There are many ways to calculate approximate values of the geometric buckling for a reactor of any shape. The following methods were used in the present work:

(a) The body is assumed to have the buckling of a cylinder with the same volume. This will give a correct answer in the limit of a very long cylindrical part, but will be a poor approximation when this part is small. The method was used for a cylinder with one end segment.

(b) The body is assumed to have the buckling of a spheroid with the same volume and radius [8]. This will also give the correct value in the limit of a very long cylindrical part. Further, it will be a better approximation when both ends are rounded. For two hemispheres it will even give the correct value when the cylindrical part vanishes, i.e. for a sphere. This method was used for a cylinder with two end segments or hemispheres.

(c) The variation method [9]. Here very good accuracy can be obtained in principle, but the amount of algebraic work imposes a limit in practice. The

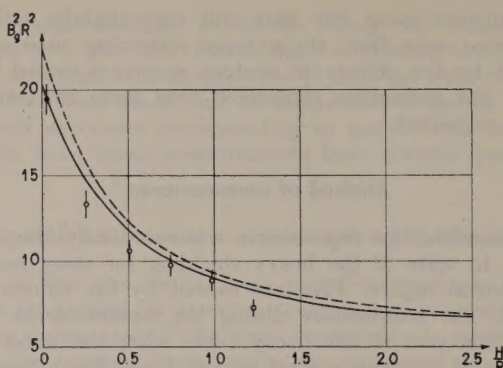


Fig. 1. $B_g^2 R^2$ for a cylinder with one hemisphere as a function of the ratio of the height of the cylinder and its radius. The full curve is calculated by machine, the dashed by a variation method and the points are measured.

method was used for the cylinder with one hemisphere and with the following trial functions (in cylindrical coordinates):

$$\text{In the cylinder} \quad \phi_1 = (R^2 - r^2) [A(H - z) + C(H - z)^2]. \quad (6)$$

$$\text{In the hemisphere} \quad \phi_2 = (R^2 - z^2 - r^2) (Dz + E). \quad (7)$$

R and H are the radius and height of the cylinder, and the origin is in the interface between the two parts of the body. The trial functions satisfy the boundary condition $\phi = 0$ on the surfaces of the body. Two relations are obtained for the four parameters from the continuity conditions at the interface. The remaining two parameters are varied to minimize the integral

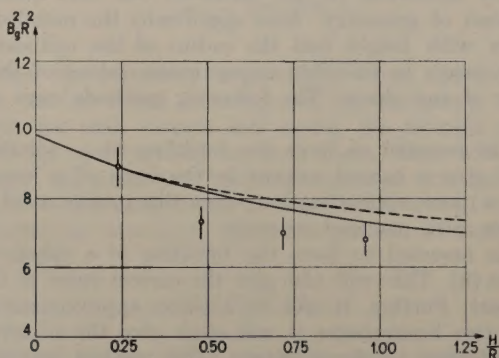


Fig. 2. $B_g^2 R^2$ for a cylinder with two hemispheres as a function of the ratio of the height of the cylinder and its radius. The full curve is calculated by machine, the dashed by a spheroid approximation and the points are measured.

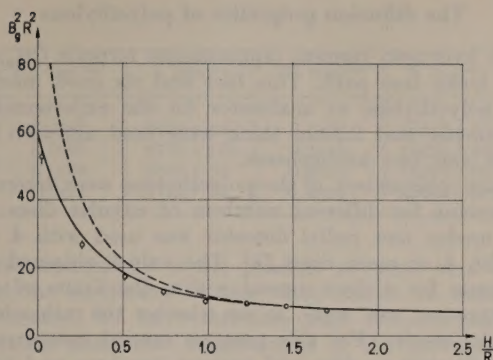


Fig. 3. $B_0^2 R^2$ for a cylinder with one end segment as a function of the ratio of the height of the cylinder and its radius. The full curve is calculated by machine, the dashed by a cylinder approximation and the points are measured.

$$I = \int_V (-\phi \nabla^2 \phi - B_0^2 \phi^2) dV. \quad (8)$$

The resulting expression for the buckling as a function of the ratio H/R is rather close to the correct one. However, in the limit of $H/R = 0$ it gives $B_0^2 R^2 = 22.5$ instead of 20.191 and in the limit of $H/R = \infty$ the value 6 instead of 5.7831.

(d) Finally the buckling was calculated with a high speed electronic computer using a code for reactor calculations in cylindrical geometries. The spherical surfaces were then approximated by step functions. The overall uncertainty was estimated at about one per cent.

The results of the different calculations are shown in Figs. 1-4 along with the experimental results.

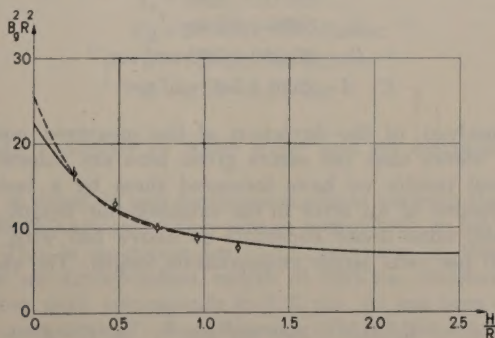


Fig. 4. $B_0^2 R^2$ for a cylinder with two end segments as a function of the ratio of the height of the cylinder and its radius. The full curve is calculated by machine, the dashed by a spheroid approximation and the points are measured.

The diffusion properties of polyethylene

Due to its high hydrogen content (approximate formula CH_2) polyethylene has a short transport mean free path. This fact and its good mechanical properties led us to choose polyethylene as moderator for the experiments. Eight circular discs 18 cm in diameter and 2.2 cm thick were used and also two spherical segments 4.5 cm high and two hemispheres.

First the diffusion parameters of the polyethylene were determined by measuring the time constant for different numbers of circular discs. To suppress the higher harmonic modes one radial detector was used with 4 or less discs and two axial ones with 4 or more discs [5]. The values obtained with the different detector arrangements for 4 discs agreed within the limits of error.

A special investigation was made to see whether the rather large BF_3 counters would influence the results. For this purpose several measurements were made using one single polyethylene disc and various numbers and types of detectors. The largest detector was made of a 3 cm diameter copper tube with 1 mm wall thickness. Assuming it to act as a thin reflector of neutrons it was expected to cause a decrease in the decay constant of 0.2 per cent when placed behind a cadmium mask. The measurements indicated the effect to be five times as large. However, because of the uncertainties in the measurements with this small geometry the result could not be regarded as significant. In the evaluation of the results a correction of 1 ± 1 per cent was assumed for one disc and correspondingly less for the larger geometries.

In calculating the buckling values the physical dimensions of the moderator were assumed to be known to within ± 0.05 mm. The extrapolation length was calculated as $0.71l_{tr}$, with $l_{tr} = 0.321$ cm. Small corrections were made for the change in the neutron spectrum caused by diffusion cooling.

The decay constants and bucklings obtained for the different numbers of discs are listed in Table 1. The uncertainties in the time constants shown there arise from counter statistics, corrections for the harmonic modes, detector disturbance, and from the uncertainties in the physical dimensions. From the errors in the decay constants weighting coefficients were calculated which were then used in a least-squares fit of the results to a parabola. The following values were obtained:

$$\begin{aligned}\lambda_a &= 5895 \pm 55 \text{ sec}^{-1}, \\ D_0 &= 26\,480 \pm 360 \text{ cm}^2 \text{ sec}^{-1} \\ C-d &= 2550 \pm 340 \text{ cm}^4 \text{ sec}^{-1}.\end{aligned}$$

A statistical analysis of the deviation of the measured points from the calculated parabola shows that the errors given here are somewhat too small. In evaluating the final results we have increased them by a factor 1.5.

To see the influence of an error in the extrapolation length, a least-squares fit was made with the same decay constants as above but with the bucklings calculated with a 10 per cent larger extrapolation length. The changes in the parameters were

$$\text{in } \lambda_a: -21 \text{ sec}^{-1}; \text{ in } D_0: +290 \text{ cm}^2 \text{ sec}^{-1}; \text{ in } C-d: -350 \text{ cm}^4 \text{ sec}^{-1}.$$

Errors of these magnitudes should therefore be taken into account in quoting final errors.

Table 1.

B_D^2 (cm ⁻²)	λ (sec ⁻¹)	B_D^2 (cm ⁻²)	λ (sec ⁻¹)
0.0981	8493 $\pm$ 40	0.1825	10517 $\pm$ 50
0.1070	8772 $\pm$ 120	0.2645	12675 $\pm$ 60
0.1206	9008 $\pm$ 50	0.4835	18220 $\pm$ 90
0.1427	9725 $\pm$ 50	1.4844	39440 $\pm$ 400

Another systematic uncertainty is the possible presence of a B^6 term in eq. (1). The magnitude of the coefficient of this term cannot be estimated at the present state of the theory. We have therefore only assumed that the B^6 term introduces the following additional uncertainties in the parameters

in λ_a : ± 20 sec⁻¹; in D_0 : ± 100 cm² sec⁻¹; in $C-d$: ± 500 cm⁴ sec⁻¹.

The final values are therefore:

$$\begin{aligned}\lambda_a &= 5900 \pm 90 \text{ sec}^{-1}, \\ D_0 &= 26\,500 \pm 600 \text{ cm}^2 \text{ sec}^{-1}, \\ C-d &= 2600 \pm 800 \text{ cm}^4 \text{ sec}^{-1}.\end{aligned}$$

The correction term d is of the order of 400 cm⁴ sec⁻¹ [1], and the diffusion cooling coefficient is therefore

$$C = 3000 \pm 800 \text{ cm}^4 \text{ sec}^{-1}.$$

From the diffusion constant a transport mean free path of 0.320 ± 0.008 cm is obtained. By combining the values of the diffusion constant and absorption rate a diffusion length of 2.12 ± 0.03 cm is obtained.

Dio [10] made similar measurements on paraffin wax. His results were

$$\begin{aligned}\lambda_a &= 5860 \pm 70 \text{ sec}^{-1}, \\ D_0 &= 26\,600 \pm 500 \text{ cm}^2 \text{ sec}^{-1}, \\ C-d &= 1\,200 \pm 500 \text{ cm}^4 \text{ sec}^{-1}, \\ L &= 2.13 \pm 0.04 \text{ cm}.\end{aligned}$$

The paraffin had a density of 0.890 g/cm³ and its composition was assumed to be C₂₅H₅₂.

The differences in densities and hydrogen contents between the polyethylene and the paraffin counterbalance each other almost completely, so his parameters should agree with ours. This is so except for $C-d$. However, in the evaluation of his data Dio assumed an extrapolation length of 0.28 cm, whereas his result for the transport mean free path corresponds to 0.23 cm. As has been pointed out above this influences the parameter $C-d$ considerably. Taking this into account his results agree with ours within the limits of error.

It should be noted that the values for the diffusion length obtained by Dio and by us disagree with earlier measurements [11, 12], which gave values about 2.4 cm.

The capture cross section of hydrogen

From the absorption probability the capture cross section of hydrogen can be calculated, provided the composition of the polyethylene is known. A careful analysis was made by the Swedish Government Testing Institute (Statens Provvningsanstalt). Three samples were analysed with results showing very high internal consistency. The averages are

Density 0.918 ± 0.001 g/cm³

Carbon content . 85.6 ± 0.1 per cent

Hydrogen content 14.4 ± 0.1 per cent

The carbon and hydrogen values were determined independently. No ash residue was found within the experimental uncertainty of ± 0.002 per cent.

The polyethylene can therefore be assumed to have a composition CH_x where $x = 2.005 \pm 0.010$. It was manufactured by polymerisation of ethylene gas under high pressure without the addition of any catalyst. Thus the probability of any significant impurity of cadmium or any other metal is considered to be negligible. However, there might be some possibility of a boron impurity and therefore a spectrographic analysis was made. The result was that the boron content is below 3 parts per million. In the evaluations we assume it to be 1.5 ± 1.5 ppm.

Using the above results and the value 3.4 mb for the capture cross section of carbon, the capture cross section of hydrogen at 2200 m/sec is found to be 0.337 ± 0.005 barns. In Table 2 the most accurate results of absolute determinations of this cross section are listed, and the agreement is satisfactory. The most recent measurement relative to boron gave 0.327 ± 0.004 barns [14].

Table 2. Absolute determinations of the 2200 m/sec capture cross section of hydrogen.

Reference	Cross section (barns)
[5] von Dardel & Sjöstrand (1954)	0.333 ± 0.003
[13] Meads <i>et al.</i> (1956)	0.335 ± 0.004
[10] Dio (1958)	0.328 ± 0.006
— Present work	0.337 ± 0.005

Buckling measurements

When the relation between time constant and buckling had been established by measurements on polyethylene cylinders, the four types of special geometries described above were then studied. They were made from the hemispheres and segments with one or more of the cylindrical discs. From the measured time constant of each geometry the corresponding buckling (corrected for the detector disturbance) was calculated using the previously determined parameters for the least-squares fitted parabola and errors given on page 476. In these measurements the uncertainty due to the presence of higher harmonic modes was somewhat larger than for the pure cylinders due to the difficulty to foresee their shape and importance.

The results of the measurements are given in figs. 1-4 where they are compared with the approximate calculations and with the machine computations. Over the wide range of bucklings the theoretical and experimental values follow each other closely. However, there is a definite tendency of the measured points to fall below the theoretical curves. This could be explained if the possible systematic errors which arise from the extrapolation length and from higher order terms in eq. (1) depend on the shape of the geometry. The deviation of the measured value for a segment without any disc might be caused by this, but it seems improbable that it could be possible for the larger geometries.

The effect of control rods

The effect of small absorbers corresponding to control rods in a reactor was also studied with the pulsed method. The absorbers are considered to change the shape of the geometry so that the geometric buckling is altered. From measurements with and without a rod the change in geometric buckling caused by a rod can be calculated and compared with theoretical predictions.

We consider a cylindrical body with effective radius R , effective height H and with a totally absorbing rod extending axially through the centre of the cylinder. The solution of the one-group equation (2) with the boundary conditions that $\phi = 0$ at the outer surfaces of the body then gives

$$\phi = A \cos \frac{\pi z}{H} \left[J_0(\alpha r) - \frac{J_0(\alpha R)}{Y_0(\alpha R)} Y_0(\alpha r) \right], \quad (9)$$

$$\text{with} \quad B_g^2 = \alpha^2 + \pi^2/H^2. \quad (10)$$

The boundary conditions at the rod (radius a) can be applied in two different ways.

(a) An effective radius a_0 is calculated at which the neutron flux is zero. We rely then on the result from transport theory that the asymptotic neutron distribution in an infinite medium in the presence of a thin, black, cylindrical absorber can be expressed as

$$\phi_{as} = C_1 + C_2 \log \frac{r}{a}. \quad (11)$$

Applying the condition that the logarithmic derivative should equal the inverse extrapolation length we find the effective radius

$$a_0 = a \exp(-\mu l_{tr}/a), \quad (12)$$

where μl_{tr} is the extrapolation length. This boundary condition gives the following relation for determination of α :

$$J_0(\alpha a_0) Y_0(\alpha R) = J_0(\alpha R) Y_0(\alpha a_0). \quad (13)$$

(b) Application of the boundary condition for the logarithmic derivative directly to the flux of eq. (9) gives the following equation for α :

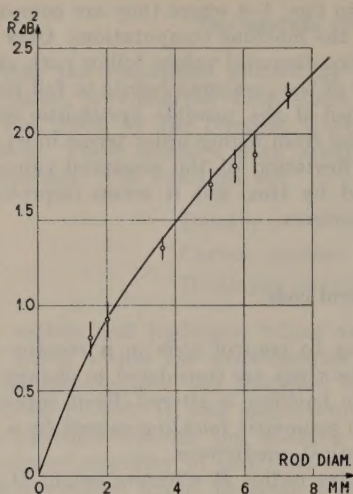


Fig. 5. $R^2 \Delta B_g^2$ for a central cadmium rod in a 18 cm water cylinder as a function of the rod diameter. The curve is calculated and the points are measured.

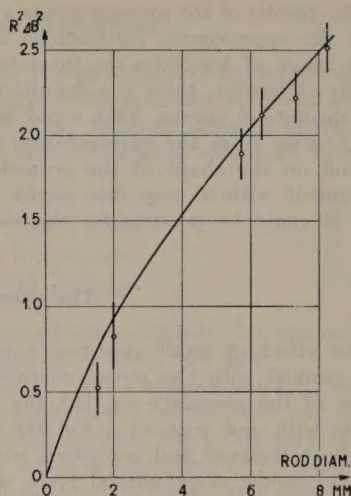


Fig. 6. $R^2 \Delta B_g^2$ for a central cadmium rod in a 14.5 cm water cylinder as a function of the rod diameter. The curve is calculated and the points are measured.

$$\alpha \mu l_{tr} = \frac{J_0(\alpha a) Y_0(\alpha R) - J_0(\alpha R) Y_0(\alpha a)}{J_0(\alpha R) Y_1(\alpha a) - J_1(\alpha a) Y_0(\alpha R)}. \quad (14)$$

In both cases μ is a function of a and of l_{tr} and is given in the *Reactor Handbook* [9]. Eqs. (13) and (14) were solved numerically. It was found that for the absorbers used in the experiment the two different methods of calculation agreed within less than one per cent. This means that expression (11) for the asymptotic neutron flux is valid. Hence the simple expression (13) can be used instead of (14).

Control rod measurements

For the control rod measurements water was used as a moderator and the absorbers were cylindrical cadmium rods ranging from 0.15 to 0.83 cm in diameter. The water was contained in cylindrical vessels 14.5 or 18 cm in diameter; the height of the water was about 15 cm.

Measurements were made with and without the rods in the water, and from the difference in decay constant, $\Delta \lambda$, the difference in buckling, ΔB^2 , was obtained as

$$\Delta B^2 = \frac{\Delta \lambda}{D_0 - (C - d)(2B^2 + \Delta B^2)} \quad (15)$$

where B^2 is the buckling before insertion of the rod. For the diffusion parameters the data obtained in reference 5 were used. Small corrections were introduced for

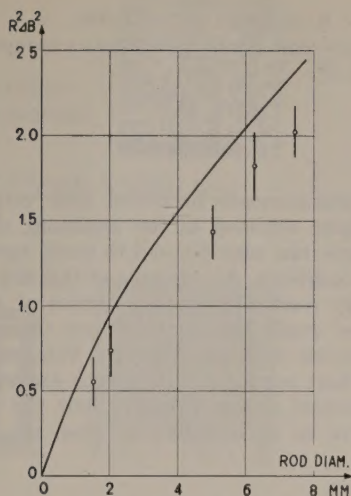


Fig. 7. $R^2 \Delta B_g^2$ for a diagonal cadmium rod in 14.5 cm water cylinder as a function of the rod diameter. The curve is calculated and the points are measured.

differences in water level due to evaporation and different volumes of the rods. The errors in the measured values are mainly caused by the uncertainty in $\Delta \lambda$.

In the first experiments the rods were placed centrally along the axis of the cylinder. The results are shown in Figs. 5 and 6 for the two cylinders used. The rather large errors in the results with the smaller cylinder were caused by background disturbances. The agreement with the theoretical predictions is good. This indicates that the commonly used theory for control rod calculations can predict the effect of thin, black rods on the thermal neutron distribution with good accuracy. So the large disagreement found by Price from measurements in a critical assembly [15] must be due to incomplete understanding of, for instance, the effect of fast neutrons or the influence of the inhomogeneity of the assembly.

A few measurements were made to study the effect of an axial control rod as a function of its distance from the centre of the cylinder. However, because of the rather large uncertainty in this kind of measurements the results could neither be said to agree nor disagree with the theoretical predictions.

The final measurements were devoted to cadmium rods suspended diagonally through the 14.5 cm cylinder. Such oblique control rods are not unusual in reactors, but their influence on the reactivity is difficult to calculate. The results of these measurements are shown in Fig. 7. The theoretical curve was calculated with the simple assumption that the effect of a diagonal control rod relative to an axial one should be equal to the ratio of the integrals of the square of the neutron flux over the rod volumes. This ratio is

$$\frac{4}{\pi} \sqrt{1 + \frac{4R^2}{H^2}} \int_0^{\frac{1}{2}\pi} J_0\left(\frac{4.81x}{\pi}\right) \cos^2 x dx. \quad (16)$$

For the present cylinder it is found to be 1.006.

As seen from the figure our crude theoretical assumptions give systematically too high values for $R^2 \Delta B^2$.

Conclusions

The results of the measurements described here can be applied to reactors according to the principles outlined in the beginning of this article. However, measurements of this type can also be used to study special problems in neutron physics under idealized conditions. An example of this are the conclusions we could draw about the boundary conditions at the surface of a thin, absorbing rod.

In this connection we would like to point out the possibility of performing analogue reactor experiments without neutrons. This has been done with optical systems [16] and using heat conduction [17]. Such experiments are in many cases simpler and the experimental accuracy higher, but the analogy with a thermal neutron system can never be so complete as when the present method is used.

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